Computational Geometric System Identification

Mahdis Bisheban, Taeyoung Lee
Flight Dynamics and Control Laboratory
Department of Mechanical and Aerospace Engineering
School of Engineering and Applied Science
The George Washington University, Washington, DC

Motivation

- Popularity of rotary-wing unmanned vehicle is increasing, because of simple mechanical structure, small size, low manufacturing price, vast ability,
- Controlling UAV in adverse weather condition is an open problem,
- To address this problem, we need to know the precise dynamic model of UAV (Identification).

Background

Attitude estimation has been studied in terms of

- Euler angles (Suffering from singularities)
- Quaternions (Challenging to represent sensitivities)
- Special orthogonal group (Using constrains or projections to avoid deviation of numerical trajectories of rotation matrices from SO(3))

Attitude dynamics of a rigid body

Attitude dynamics of rigid body :

$$J\dot{\Omega} + \Omega \times J\Omega = M_c,$$
$$\dot{R} = R\hat{\Omega},$$
$$\mathrm{SO}(3) = \{R \in \Re_{3\times 3} \,|\, R^T R = I_{3\times 3},\ \det[R] = 1\}$$

A Lie group variational integrator:

$$h(J\Omega_k)^\wedge = F_k J_d - J_d F_k^T,$$
$$R_{k+1} = R_k F_k,$$
$$J\Omega_{k+1} = F_k^T J\Omega_k + hM_{c,k+1},$$

Method

Problem formulation: The goal is to estimate the inertia matrix $J(\boldsymbol{\theta})$ such that estimated trajectory $\{(R(t), \Omega(t)\}$ is consistent with the given input-output trajectory $\{(R_z(t), \Omega_z(t), M_z(t)\}$, while satisfying inequality constraints $c_j(\boldsymbol{\theta}), j = 1, 2, 3$ imposed by positive definiteness of J.

$$J(\boldsymbol{\theta}) = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \\ \theta_2 & \theta_4 & \theta_5 \\ \theta_3 & \theta_5 & \theta_6 \end{bmatrix}$$

Constraints:

$$c_1(\boldsymbol{\theta}) = \theta_1 > 0,$$
$$c_2(\boldsymbol{\theta}) = -\theta_2^2 + \theta_1\theta_4 > 0,$$
$$c_3(\boldsymbol{\theta}) = -\theta_6\theta_2^2 + 2\theta_2\theta_3\theta_5 - \theta_4\theta_3^2 - \theta_1\theta_5^2 + \theta_1\theta_4\theta_6 > 0.$$

Cost function:

$$C(\boldsymbol{\theta}) = \frac{1}{2N}\sum_{k=1}^{N}\{\frac{1}{a_1}\tilde{\Omega}_k^T\tilde{\Omega}_k + \frac{1}{a_2}\mathrm{tr}[I_3 - \tilde{R}_k]\}.$$

Errors:

$$\tilde{R}_k = R_{z_k}^T R_k, \quad \tilde{\Omega}_k = \Omega_{z_k} - \Omega_k.$$

Necessary conditions for optimality:

$$\delta C_a(\boldsymbol{\theta}) = \delta C(\boldsymbol{\theta}) + \sum_{j=1}^{m}\lambda_j \delta c_j(\boldsymbol{\theta}) = 0,$$
$$\lambda_j c_j(\boldsymbol{\theta}) = 0, \quad c_j(\boldsymbol{\theta}) \geq 0, \quad \lambda_j \leq 0.$$

Perturbation model on SO(3): we propose an intrinsic formulation with exponential map:

$$R_k(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}) = R_k(\boldsymbol{\theta})\exp(\hat{\eta}_k(\boldsymbol{\theta} + \Delta\boldsymbol{\theta})),$$

where $\eta_k : \Re^p \to \Re^3$, p is the number of unknown parameters. So perturbation is given by

$$\frac{\partial R_k(\boldsymbol{\theta})}{\partial\theta_i} = R_k(\boldsymbol{\theta})\frac{\partial\hat{\eta}_k(\boldsymbol{\theta})}{\partial\theta_i},$$
$$\frac{\partial F_k(\boldsymbol{\theta})}{\partial\theta_i} = F_k(\boldsymbol{\theta})\frac{\partial\hat{\zeta}_k(\boldsymbol{\theta})}{\partial\theta_i}.$$

Output Perturbation:

$$\frac{\partial\eta_{k+1}}{\partial\theta_i} = \{R_{k+1}^T(R_k\frac{\partial\hat{\eta}_k}{\partial\theta_i}F_k + R_kF_k\frac{\partial\hat{\zeta}_k}{\partial\theta_i})\}^\vee,$$
$$J\frac{\partial\Omega_{k+1}}{\partial\theta_i} = -\frac{\partial\hat{\zeta}_k}{\partial\theta_i}F_k^T J\Omega_k + F_k^T J\frac{\partial\Omega_k}{\partial\theta_i} + F_k^T\frac{\partial J}{\partial\theta_i}\Omega_k - \frac{\partial J}{\partial\theta_i}\Omega_{k+1} + h\frac{\partial M_{c,k+1}}{\partial\theta_i},$$
$$\frac{\partial\zeta_k}{\partial\theta_i} = F_k^T(\mathrm{tr}[F_kJ_d]I_{3\times 3} - F_kJ_d)^{-1} \times\{h(J\frac{\partial\Omega_k}{\partial\theta_i} + \frac{\partial J}{\partial\theta_i}\Omega_k)^\wedge - F_k\frac{\partial J_d}{\partial\theta_i} + \frac{\partial J_d}{\partial\theta_i}F_k^T\}^\vee.$$

Variation of cost function:

$$\delta C(\boldsymbol{\theta}) = \sum_{i=1}^{p}\{\sum_{k=1}^{N}(-\frac{1}{a_1}(\frac{\partial\Omega_k}{\partial\theta_i})^T\tilde{\Omega}_k - \frac{1}{2a_2}\mathrm{tr}[\tilde{R}_k\frac{\partial\hat{\eta}_k}{\partial\theta_i}])\}\frac{\delta\theta_i}{N}.$$

Numerical examples

Initial error $\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_{exact}\|$	Estimation error $\|\boldsymbol{\theta} - \boldsymbol{\theta}_{exact}\|$	Number of Iterations
1.88	3.8×10^{-2}	45

Table 1: Simulation results for $\boldsymbol{\theta}_{exact} = [1, 0.1, 0.2, 3, 0.3, 2]^T$

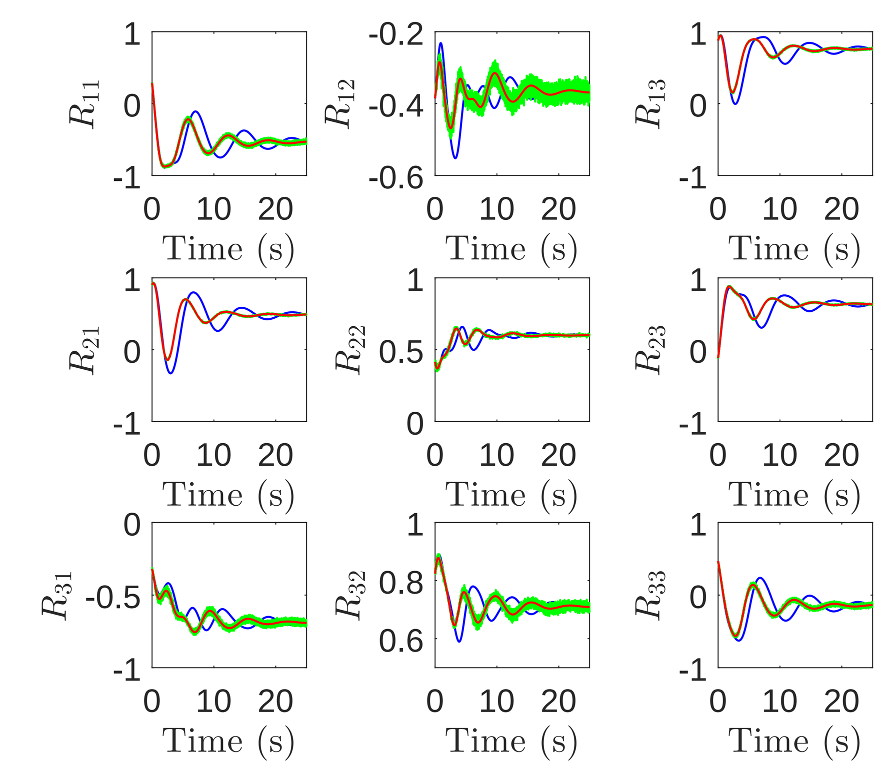

Figure 1: Attitude, (reference:green, initial:blue, estimated:red)

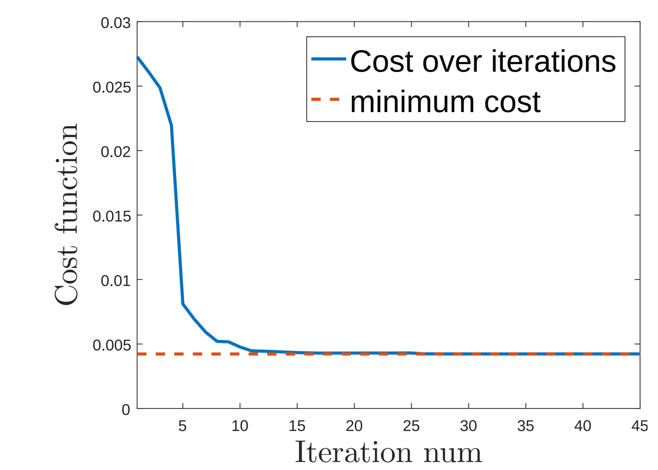

Figure 3: Cost function $C(\boldsymbol{\theta})$

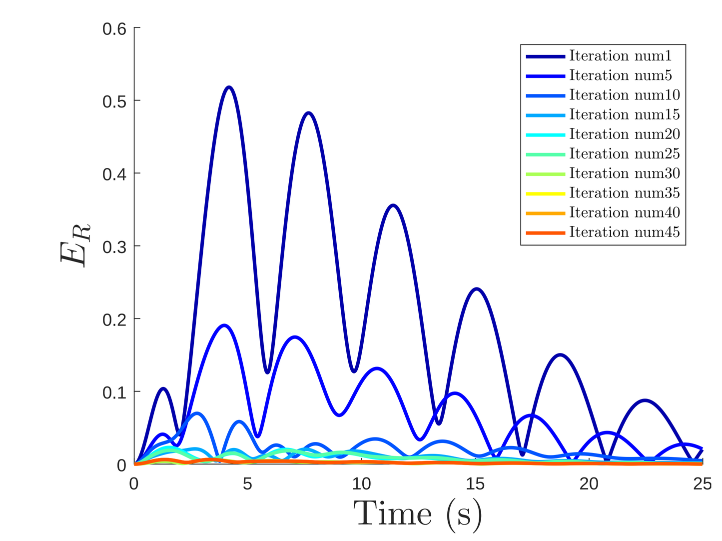

Figure 4: Attitude error $\|I_{3\times 3} - R_{z,k}^T R_k\|$

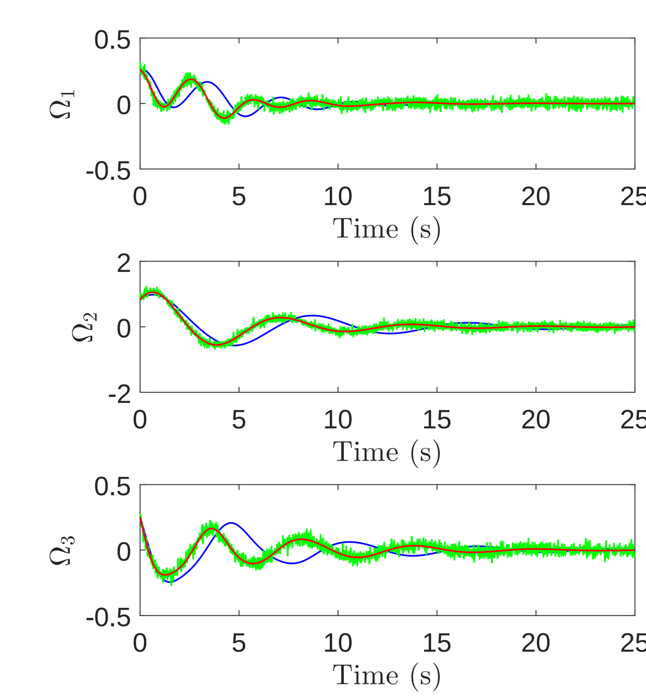

Figure 2: Angular velocity (rad/s)

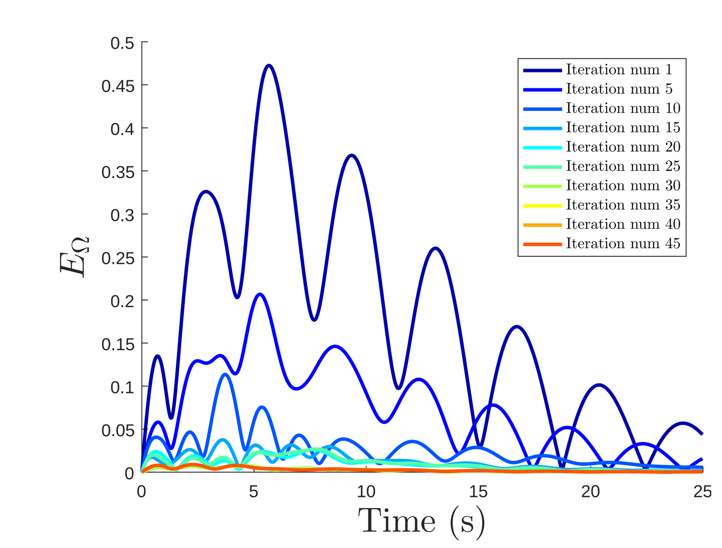

Figure 5: Angular velocity error $\|\Omega_{z,k} - \Omega_k\|$

Conclusion and future research

- Identification problem is formulated as a constrained optimization problem,
- cost function is defined as the discrepancies between the reference and simulated trajectories,
- constraints are imposed to satisfy the properties of the unknown parameters,
- attitude is represented on SO(3),
- discrete attitude dynamics are represented by Lie group variational integrator to preserve attitude on SO(3),
- perturbation model is constructed directly on the tangent space of SO(3),
- discrete-time necessary optimality conditions are constructed as variation of the cost function considering the constraints,
- proposed method can be applied to estimate of any unknown parameter of the attitude dynamics of the rigid body, e.g. blade flapping angle, and drag coefficients